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ABSTRACT

After reviewing the basic principles of instructional design, the author outlines a seventeen-step procedure for the development of mathematics lessons for primary students. The procedure consists of three basic components. The first is concerned with the definition of the unit, and steps in it concern the stating of objectives and analysis of tasks. The second component involves specifying the necessary steps in a lesson. Steps in this process begin with designing criterion items for the terminal objective and, working backwards, repeating this process for prerequisite skills until the skills defined should have been learned in a previous lesson. The third component provides for the sequential development of lesson exercises. The procedure has been used to advantage in the development of mathematics lessons for kindergarten and first-grade pupils in an individulized mathematics program. (SD)

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A Specification of Steps to be Followed in the Design of Primary Grade Mathematics Lessons for Use in Individualized Instruction

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1976

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A SPECIFICATION OF STEPS TO BE FOLLOWED IN THE DESIGN OF PRIMARY GRADE MATHEMATICS LESSONS FOR USE IN INDIVIDUALIZED INSTRUCTION

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The design of lesson materials for any subject-matter area is a creative activity which depends heavily upon the ingenuity and artistry of the designer. However, in large scale development projects where a number of persons will be involved in the actual design of lessons there is some need for a common set of guidelines for design so that the lessons produced have a certain degree of uniformity. This uniformity is particularly important in individualized systems where there is a heavy reliance ypon independent study and where lesson materials should be largely self-instructional. This paper reports on the development and application of a set of guidelines for lesson design which were derived from established learning principles and used in the design of primary grade mathematics lessons.

The specific development task for the project was to design mathematics lessons for use in kindergarten and first grade in a program stressing individualized instruction. The need was for lessons that could be used largely in independent study and that required very little reading. Mathematical content to be taught included certain basic concepts and an introduction to the fundamental operations of addition and subtraction. The authors have had previous experience in developing both manipulative and paper-and-pencil lessons of the general type required. (Lindvall,

Light, and LaPresta, 1975) In the present project they were faced with the task of developing step-by-step procedures that could be used on an independent basis by several staff members working on a group of paper-and-pencil lessons and that could be used effectively by possible new employees. The authors were also seeking step-by-step procedures which would result in lessons requiring fewer modifications after pupil tryouts. A reduction in the amount of time and effort that must be invested in lesson tryout and revision can be of critical importance on projects where large numbers of lessons must be produced.

Basic Definitions and Relevant Principles

To identify the learning principles that were likely to be of most value in this work it was necessary to specify the type of learning involved and to search for the principles that have been formulated for that form of learning. The following sections provide specific definitions of learning and instruction and then list the relevant principles associated with these definitions or approaches.

Learning and Instruction

For the type of lesson that was the focus of this study learning is assumed to be the acquisition of the capability of displaying a specified behavior in response to a given stimulus situation. This definition is derived largely from that given by Gagné.

> Learning is a change in human disposition or capability, which can be retained, and which is not simply ascribable to the process of growth. The kind of change called learning exhibits itself as a change in behavior, and the inference of learning is made by comparing what behavior was possible before the individual was placed in a "learning situation" and what behavior can be exhibited after such treatement (Gagné, 1965, p. 5)

In general terms, instruction may be defined as the process of facilitating learning. Corey defines instruction somewhat more completely as the:

> process whereby the environment of an individual is deliberately manipulated to enable him to learn to emit or engage in specified behaviors under specified conditions or as responses to specified situations (Corey, 1967, p. 6).

Basically, the approach to instruction exemplified by the project's paper-and-pencil lessons was an application of the essential ideas of programed instruction. Skinner defines programing as:

> ...the construction of carefully arranged sequences of contigencies leading to the terminal performances which are the object of education. The teacher begins with whatever behavior the student brings to the instructional situation; by selective reinforcement he changes that behavior so that a given terminal performance is more and more closely approximated (Skinner, 1965, pp. 6 - 7).

The design of paper-and-pencil lessons of the type used in this project can be considered as an effort to apply this "principle of programing" (Holland, 1960).

Principles of Learning and Instruction

Certain basic principles of learning and instruction, derived initially from laboratory studies have been found useful in developing instructional materials (Holland, 1960 p. 275-287).

Principle I. A pupil acquires those behaviors that are reinforced (Holland, 1960, p. 276). This implies that provision must be made for the reinforcement of a correct response. It also suggests the importance of knowing exactly what behavior is being reinforced, of knowing what the pupil is doing to make a correct response.

Principle 2. To be reinforced, a behavior must be emitted (Holland, 1960, p. 278). A lesson must provide for an active response on the part of the student. This response must be a product of the desired behavior.

Principle 3. The emitting of a desired behavior can be produced through prompting. Prompts may be built into the lesson through such devices as dotted lines to be filled in or brief written instructions. Prompts, in the form of instruction, may also be provided orally by the teacher or by tape.

Principle 4. Prompts must be faded so that the pupil acquires the capability of making the desired response to the appropriate stimulus (Holland, 1960, p. 279).

Principle 5. A carefully planned, gradual progression of pupil behaviors can be used to teach relatively complex capabilities with a minimum of pupil errors (Holland, 1960, p. 279). Within a lesson, the pupil should progress through a sequence of exercises, each of which builds on the capability taught in the preceding exercise.

Principle 6. The observing behavior of the student must be controlled so that making a correct response is contingent upon the proper behavior as a response to the appropriate stimulus. ("...behavior which must occur in order to teach an answer is considerably more likely to be learned than behavior not so controlled by the design of the item" (Holland, 1965, p. 81)) or ("Since the answer is designed to assure occurrence of the appropriate behavior, it is important that the correct answer be given only after meeting the intended contingency." (Holland, 1965, p. 84)). Provision must be made for forcing the student to attend to the desired stimulus if he is to make the correct response. One approach to this is to include in a new exercise, examples from previous exercises which are similar to the examples in the new exercise but have a slightly different stimulus.

Principle 7. To teach a concept (an abstraction) the pupil must respond to many examples, each of which is an exemplar of the concept but which represents one of many possible variations in other properties (Holland, 1960, p. 282).

Specific Steps in Lesson Design

The foregoing principles and definitions were used as the basis, or justification, for the specification of seventeen steps to be followed in the design of lessons. After these steps had been formulated and revised on the basis of extended trial by project staff members, they were tested by determining if they could be used by new staff members to develop effective lessons. The finalized steps are discussed under three major sections: (A) Defining the unit, (B) Specifying needed steps in a lesson, (C) Designing lesson exercises. In the following presentation of these steps their use is illustrated through their application in the analysis of a unit in addition and in the design of one particular lesson in that unit.

A. Defining the Unit

STEP 1: Obtain a general statement of the basic goal(s) of instruction of the unit involved.

In developing lessons that can be used in self-instruction it is important that the lesson writer know exactly what abilities a pupil can be expected to have acquired in lessons studied prior to the one being designed and exactly what capabilities the pupil must acquire from this lesson in order to be prepared for subsequent ones. The writers' experience suggests that this can best be done by organizing the sequence of lessons in terms of units of study, each defined by a certain number of specific objectives. The lesson designer, as a person vitally concerned with the exact specification of the abilities taught in each unit and objective must play a key

role in this task of specification. This will start with some general statement of the goal of a given unit. In the unit being used as an example here the goal statement was: "The student will be able to write addition sentences for problems involving sums less than 10."

STEP 2. Identify the mathematical content upon which the unit of instruction is based.

This unit is intended to introduce the student to the basic meaning of the concept of addition and is based on the following generally accepted definition:

> The addition of the whole numbers a and b is the assignment of the whole number c to the ordered pair (a, b) such that N(A) + N(B) = N (AUB) where A and B are finite sets, $A \cap B = \emptyset$ and N(A) = a, N(B) = b, and N(AUB) = c. That is, a + b = c. (McFarland and Lewis, 1966, p. 108).

It can be seen that the objectives in this unit should require the student to carry out the process described in this definition, namely that addition is the union of two sets. Thus the objectives for this unit must require a response from the student which ensures that the appropriate process has been used in obtaining the solution to the problem.

STEP 3: State the terminal objective for the unit involved.

This terminal objective should describe the specific stimuli the student is to respond to as well as the specific responses required. The major reasons for requiring the exact specifications of the stimuli and responses for the instruction can be found in Principles 1 and 6.

The terminal objective specified for this unit was:

> Objective M8C Given the directions "Add x and y", "Draw the sets", and "Write the number sentence", the pupil will draw a picture of a set of x elements and a set of y elements and will write the correct addition sentence. (Sum < 10)

It should be noted that this is a specific objective that the student is expected to master as a result of studying a paper-and-pencil lesson. Work on this type of lesson would typically follow students mastery of the basic operation of addition through the use of manipulative materials; that is, counting out a set of objects to represent each addend, joining the two sets, and counting the total number in this union set. Drawing the two sets in this lesson, then, involves an additional example of the concrete representation of addends and of the process of finding the total in the union set. Note, too, that the objective involves the ability to generate an addition sentence given the instructions to "Add", not merely the ability to give an answer to an addition "fact". The mastery of this objective could be an early step in the students' acquiring an understanding of the concept of addition (as defined in step 2) and of the relationship between the process of forming a union of two sets and the written addition sentence that represents that process.

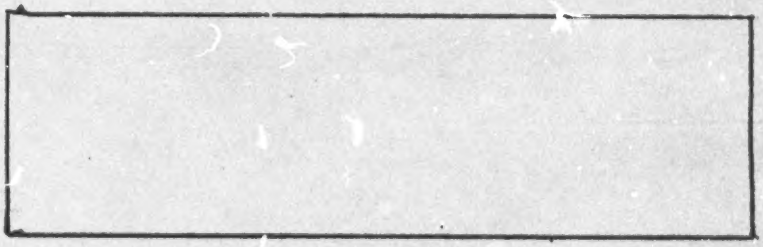
STEP 4: Develop an example of the terminal behavior to be taught in the unit.

No matter how clearly the terminal objective is defined, the lesson developer should begin by translating the objective into an actual criterion item. At this time, the lesson developer translates the desired stimuli into an actual problem format and designs a place for observing each required response. In designing the format of this criterion example the lesson designer is attempting to ensure that Principle 6 can be applied. That is, if making the desired response is to be contingent upon the proper behavior as a response to the appropriate stimulus, it is essential, first of all that the exact stimulus materials be carefully specified.

The following is an example of the current unit:

Add 5 and 4

Draw the sets.



Write the number sentence.

All criterion items for this objective are of the form shown above: (1) the addends are always given (their sum is to be less than 10), (2) the pictures of the two sets are to be drawn within the large rectangle provided, and (3) the addition sentence is to be written in the lower rectangular space. The development of the specific format for the criterion items provides rather exact direction for what is to be taught in the preceding instructional materials.

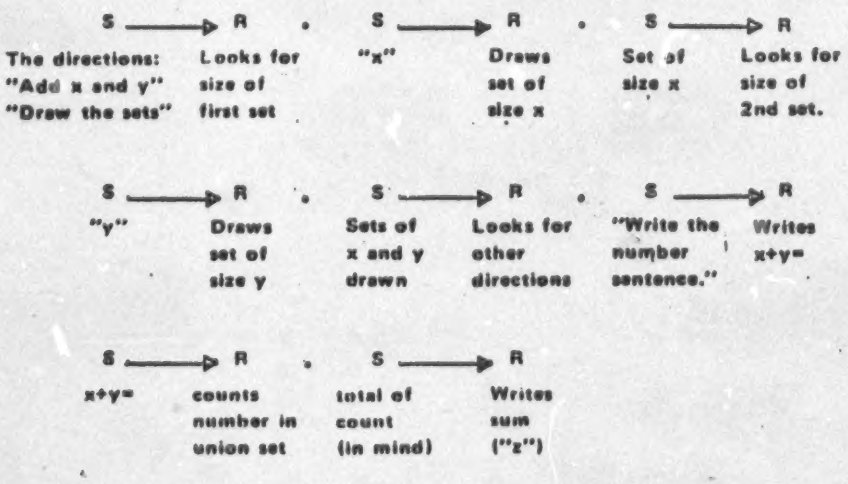
Note that this lesson is not concerned with teaching the student such things as when to apply addition, how to check addition problems, or the relationship between addition and subtraction. Such abilities are taught in lessons for other objectives. The current lesson is to teach the student to respond to the directions "Add" and "Draw the sets" so that correct answers will be produced when the student is told to add or when he or she reaches the decision in a problem situation that "I must add these numbers." The basic task of any lesson is one of being able to make a correct response to a given stimulus. However this is not to be a response based on rote memory (this may be developed later), but it is to be a response involving a representation of each addend by a set, determining the total number in the union set, and writing the addition sentence showing

the problem and its correct answer. In essence, it is to be a response involving the pupil's ability to apply the basic definition of addition. However, since the fundamental skills involved are those of making correct responses to specific stimuli, it is useful to analyze the task in terms of stimuli and responses involved and then to arrange instructional exercises which facilitate pupil acquisition of these.

STEP 5: Develop a "task analysis" for the behavior represented by the terminal objective(s)

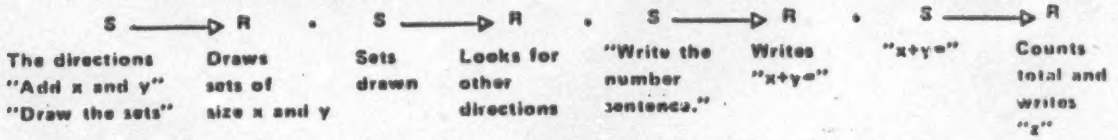
A task analysis involves the identification, in considerable detail, of all the steps that must be carried out by a pupil in performing a given task and the sequencing of these steps in the order followed. Procedures for carrying out a task analysis (or behavioral analysis or component analysis) are described by Gilbert (1962), Mechner (1967), and Resnick, Wang, and Kaplan (1973).

The analysis used here represents the presumed sequence of steps followed by the person who has mastery of the objective that is the focus of this unit. The analysis is described in terms of a chain of stimulus-response relationship where the response to a given stimulus becomes the stimulus for the next response.



STEP 6: Revise the task analysis developed in step 5 by combining any series of steps that can be taught as one process and use this to identify the unit objectives.

By combining series of S-R steps that can be taught as one process, the lesson developer can begin to identify objectives for the unit. Although this is a "first pass" at identifying objectives, programmed instruction designers recommend that first drafts of lessons should teach the largest unit of instruction possible. (Gilbert, 1962) This first draft of objectives can always be revised during the try-out stage by either combining objectives or adding others. For the unit being discussed, the revision of the task analysis developed in step 5 involved combining the first four S-R steps into one and the last two S-R steps into one, resulting in the following:



Here the original first four steps have been combined into one because the student has already learned to draw one set of an indicated size and the only new ability to be learned is that of drawing two sets of indicated sizes in response to the specific directions "Add x and y" and "Draw the sets." The last two S-Rs are combined because of the fact that counting the total and recording it as a sum is essentially one task when it is carried out in conjunction with a paper and pencil lesson.

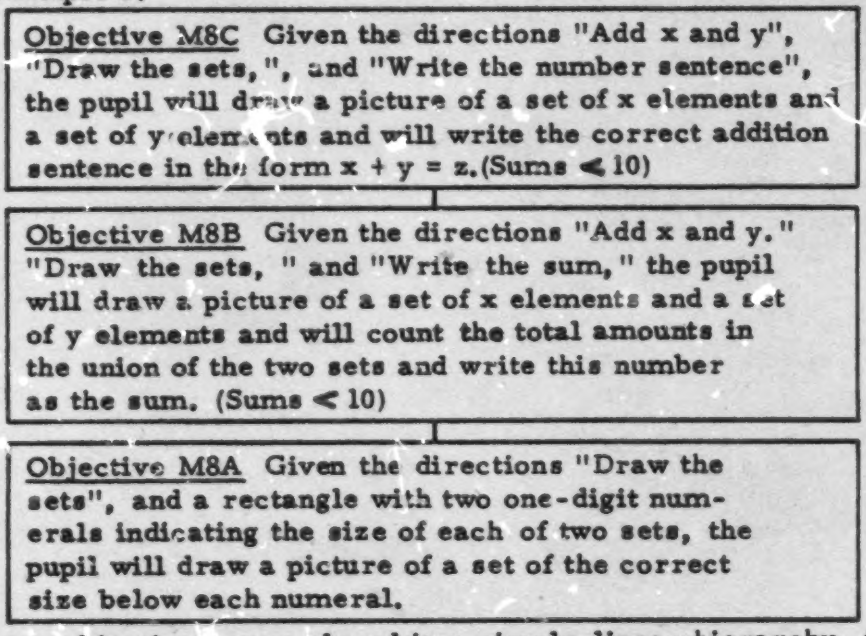
On the basis of the revised task analysis developed here a total of three objectives were identified for this unit. One of these was the terminal objective, already defined in Step 3. A second objective was based on the first S-R step in the revised analysis and involved the ability to draw the two needed sets in response to the directions "Add x and y" and "Draw the sets." A third objective combines looking for other directions with the task of counting the total number in both sets to find the sum. Note that this leaves the third S-R, writing the major part of the addition sen-

tence (x + y =), and the fourth S-R, writing the sum in the proper place in the addition sentence, to be taught as the terminal objective.

The foregoing analysis resulted in this unit consisting of the three objectives listed in Step 7.

STEP 7: Organize the objectives for the unit into a hierarchical structure based on the supposed prerequisite relationships, as indicated by the task analysis.

This hierarchical structure of objectives serves to identify the specific lessons that must be taught (typically a separate lesson for each objective). Also, it helps inform the lesson designer concerning what abilities the pupil will bring to a new lesson and for what "next lesson" the current one must prepare the student. This facilitates the application of Principle 5.


Objective M8C Given the directions "Add x and y", "Draw the sets,", and "Write the number sentence", the pupil will draw a picture of a set of x elements and a set of y elements and will write the correct addition sentence in the form x + y = z. (Sums ≤ 10)
Objective M8B Given the directions "Add x and y." "Draw the sets," and "Write the sum," the pupil will draw a picture of a set of x elements and a set of y elements and will count the total amounts in the union of the two sets and write this number as the sum. (Sums ≤ 10)
Objective M8A Given the directions "Draw the sets", and a rectangle with two one-digit numerals indicating the size of each of two sets, the pupil will draw a picture of a set of the correct size below each numeral.

These objectives were placed in a simple linear hierarchy, following the standard convention with the terminal objective at the top, and M8A being pre-requisite to M8B which, in turn, is pre-requisite to M8C. [1]

[1] For purposes of this paper this unit has been limited to these three objectives. In the actual unit they are part of a unit of seven objectives, including three involving subtraction.

After completing the first seven steps, the lesson developer has a listing of the objectives to be taught in a unit as well as a structure indicating prerequisite relationships.

B. Specifying Needed Steps in a Lesson

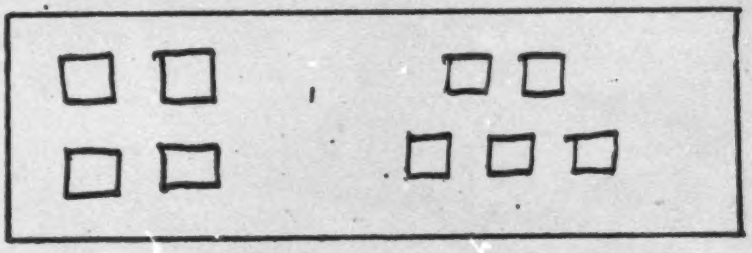
The next group of steps should be carried out in planning the lesson for each objective in the unit. For illustrative purposes the authors have chosen objective M8B as the example.

STEP 8: Select the specific objective which this lesson is to teach and develop exercises or test items that represent the criterion capability to be taught in this lesson.

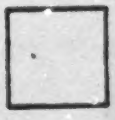
The type of test item that represents the criterion of mastery for objective M8P is shown below. An acceptable correct answer is indicated.

Add 4 and 5

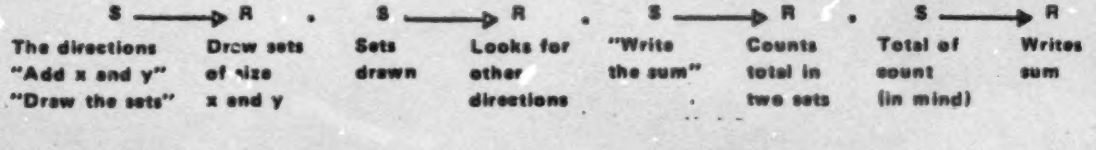
Draw the sets.



Write the sum.



STEP 9: Develop a task analysis for the capability to be taught in this lesson.


S → R
The directions "Add x and y" "Draw the sets"
Drew sets of size x and y
S → R
Sets drawn
Looks for other directions
S → R
"Write the sum"
Counts total in two sets
S → R
Total of count (in mind)
Writes sum

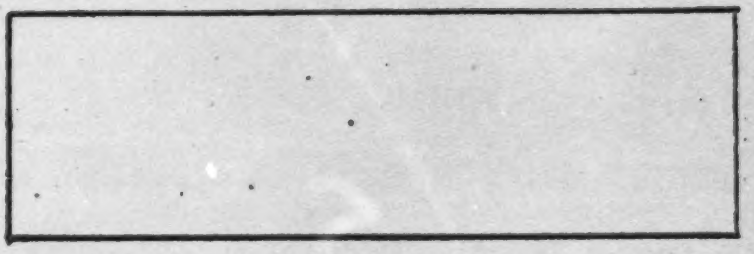
Since the instruction to be provided in lesson M8B can be based on the assumption that pupils will have mastery of the skill taught in M8A, the first step in the task analysis here is the same as the first step in the revised task analysis developed in step 6. In M8A the pupil learned to draw two sets in response to the directions "Draw the sets" when the numerals showing the set sizes were located just above the rectangle in which the sets were to be drawn. In M8B the pupil must first be taught to draw the sets when the set sizes are to be found in the sentence "Add x and y". Then the pupil learns to look for other directions, count the total number of objects in the two sets, and write the sum.

STEP 10: Develop an example of the criterion test item for the lesson and an example of an item that represents that "last thing to be taught" before the criterion item is taught. That is, develop examples of the last two steps in the lesson.

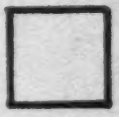
Example of criterion item:

Add 3 and 1

Draw the sets.



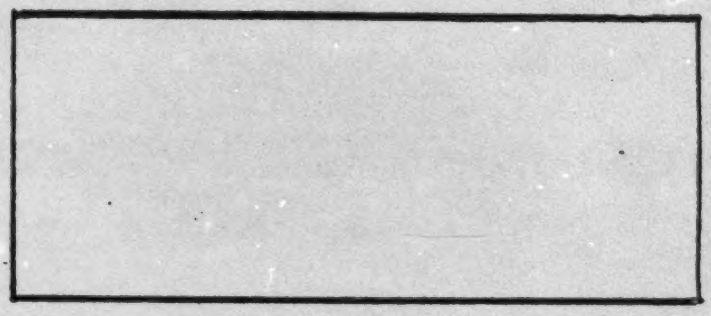
Write the sum.



Example of item to be taught as immediate prerequisite to teaching criterion item ("last thing to be taught" before criterion item):

Add 6 and 2

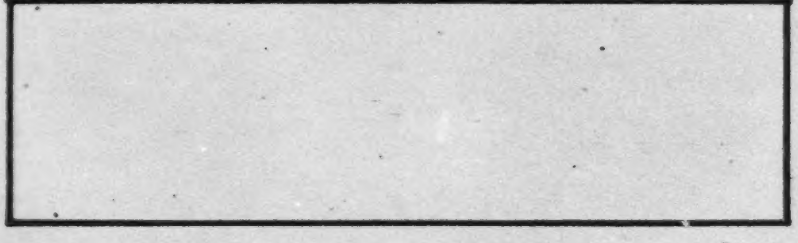
Draw the sets.



STEP 11: Repeat the process described in step 10 to develop an example of an item to be taught as the immediate prerequisite to the last item developed there.

Add 4 and 5

Draw the sets.



In this exercise, the pupil is introduced to the directions "Add x and y" for the first time. Here these directions are placed directly above the rectangular box since in lesson M8A he learned to draw sets whose size was indicated by numerals placed in this position. In the actual exercise, these directions were moved from directly over the rectangle to the left side of the page (as found in the criterion item for the objective), in

small steps.

STEP 12: Repeat the process described in step 10, to identify successive earlier steps in the lesson until the item developed is one that the pupil should have mastered in a preceding lesson.

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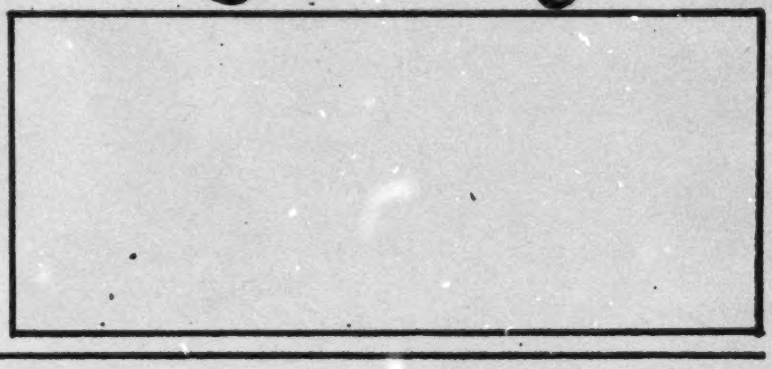
5 3

Draw the sets.



This last item represents the skill taught in lesson M8A and, therefore, the identification of steps in lesson M8B would stop with the items developed in step 11, in this case, Lesson M8B, then, will involve three basic steps, the first one involving the teaching of the ability represented by the item develop in Step 11 and two others involving the two skills specified in Step 10. Each of these 3 item types represents a criterion ability for an "exercise" within the lesson. Each such exercise is a mini-lesson which must be designed to teach the student to make the given correct response to a specific stimulus. The sequence of such exercises, in turn, makes up the complete lesson.

C. Designing Lesson Exercises

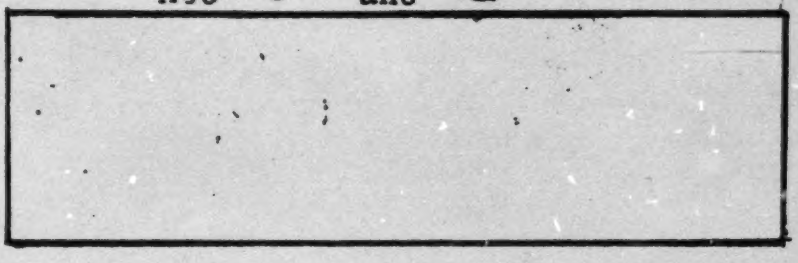
At the completion of step 12, the lesson designer has an example of each criterion exercise for a lesson. The following steps outline lesson development procedures which provide specific suggestions for designing the initial version of each exercise within a lesson.

STEP 13: Starting with the first item to be taught, develop one example of it.

The exercise used as an example here is the first exercise in M8B, the item for which was developed in step 11.

Add 3 and 2

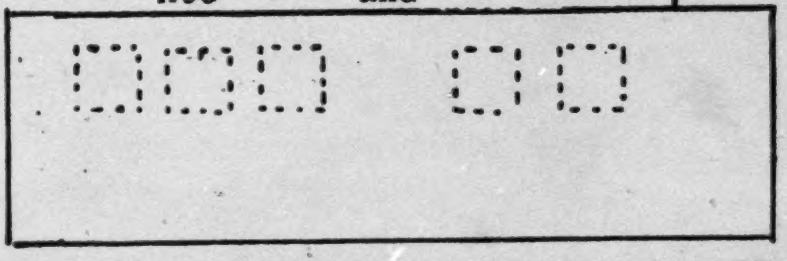
Draw the sets.



STEP 14: Ask yourself, "What minimal but sufficient prompting will cause the pupil to perform the correct behavior and make the desired response to this criterion item?" Incorporate this prompting into the item developed in step 13 and use this as the first item in the exercise.

Add 3 and 2

Draw the sets.



The new ability that the student is to acquire in this specific exercise is that of drawing the two sets in response to the stimuli "Add x and y" and "Draw the sets." The prompting consists of the dotted-line drawing of the proper sets. The goal in providing a prompted example is to cause

the pupil to relate the response to the directions and other essential stimuli of the item. The pupil should be thinking "These directions mean that I should do this (as exemplified by the prompted response." Prompting with dotted lines such as that used in this exercise may function in two different ways. Some students may "think about what they are doing" while filling in the dotted responses and may learn what the directions mean from this. Other students may merely follow the dotted lines without relating their action to the directions. For such persons, the prompted example, can however, serve the same purposes as a fully worked example that the pupil is "to study." This means that the prompted example is really a prompt for the example that follows. It can function in this way if the pupil is taught that when he or she comes to an unworked example the way to learn what to do is to study what was done in the prompted example.

STEP 15: Develop a second (plus third, etc.) prompted item of the same type as that developed in step 13 and 14 if the one prompted item does not completely illustrate the "rule" relating the behavior required in making the correct response to the defined stimulus.

In the sample exercise used here it would be useful to have a second prompted exercise, involving different numbers so that the pupil could indeed see the relationship between the given numbers and the size of each set.

It should be pointed out here that the writers do not assume that all pupils will be able to master an exercise such as this one completely on their own. Evidence shows that some pupils will. Other pupils will need teacher assistance. However, such assistance should take the form of directing the pupil's attention in such a way that he or she can learn from the exercise. The basic process involved in this learning is that of making inferences from the prompted examples. In the examples used here the student should learn to ask him/herself, "Why should I

draw this many boxes ?" The examples provide the opportunity for learning the answer to such key questions. Some students will learn such answers on their own. Others will need some form of teacher guidance.

STEP 16: Develop additional items "fading" the prompts used in the first items.

In the exercise used in the preceding example, the fading was abrupt in that items developed in Steps 14 and 15 were followed by examples of the criterion exercise, with no prompts. Additional prompted exercises were found not to be needed during the lesson try out. If they had been, then additional exercises would have been developed.

STEP 17: Provide items with no prompts as practice on a great enough variety of items to span the relevant variations in stimulus material e.g., in the numbers involved, size of sets, etc.

The theoretical basis for this step is Principle 7, namely that many exemplars of the concept or process are presented so that the pupil acquires true mastery. This also decreases the chances that a pupil may have developed his/her own inappropriate rule for responding.

In the present example, several additional items, similar to the criterion for each exercise, were developed. These items involved such paired addends as 6 and 0 (the use of zero as an addend), 4 and 4 (the two addends were the same), 4 and 5 (the first addend was a smaller number) and 8 and 1 (the first addend was the larger number).

Summary of Application of Steps

Steps 13 through 17 outline the procedure to be followed in designing each exercise within a lesson. These steps would be applied to each of the sequential items identified in steps 10, 11, and 12 to develop a teaching exercise for each such item. This sequence of exercises then constitutes the total lesson for the given objective, and when it is assembled in this way, it is ready for final editing and initial tryout with students.

Results

This investigation resulted in the development of a brief manual for lesson design which identifies seven principles of learning and instruction and which specifies seventeen steps to be followed in designing paper-and-pencil lessons for use by kindergarten and first-grade pupils in an individualized mathematics program. Information concerning the usefulness of these steps in the actual design of lessons was derived from their application in the design of several math lessons by three regular members and one temporaty member of the project staff and in the design of five lessons by one new staff member. These steps were shown to be of value in permitting staff members, each working on some portion of the lesson materials required for a given program, to design lessons with comparable characteristics and format. The lessons produced in this way were of demonstrated effectiveness in try-outs with students and required fewer modifications after undergoing a preliminary tryout than had been the case with lessons designed following earlier procedures.

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